N O T I C E

THIS DOCUMENT HAS BEEN REPRODUCED FROM MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE

THE UNIVERSITY OF ALABAMA
COLLEGE OF ENGINEERING
BUREAU OF ENGINEERING RESEARCH

(NASA-CR-1?3387) DEVELOPMENT OF MATHEMATICAL MODELS FOR SOLID STATE SWITCHING DEVICES Final Report (Alabama Univ., University.) 30 p HC A03/MF A01 CSCL 09A G3/33

N80-19426

Unclas 47432

FINAL REPORT

on

CONTRACT NO. NAS8-33430

DEVELOPMENT OF MATHEMATICAL MODELS FOR SOLID STATE SWITCHING DEVICES

Prepared for

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Marshall Space Flight Center, AL 35812

Prepared by

W. D. Raburn, Principal Investigator
Associate Professor of Electrical Engineering
P. O. Box 6169
The University of Alabama
University, Alabama 35486
and
J. C. Kim, Research Associate

Submitted by

The University of Alabama
P. O. Box 2846
University, Alabama 35486

January, 1980

BER REPORT NO. 24?-25

RECEIVED NASA STI FACILITY ACCESS DEPT.

UA ...UNIVERSITY, ALABAMA 35486

BUREAU OF ENGINEERING RESEARCH

Members of the faculty who teach at the undergraduate and graduate levels, along with their graduate students, generate and conduct the investigations that make up the College's research program. The College of Engineering of The University of Alabama believes that research goes hand in hand with teaching. Early in the development of its graduate program, the College recognized that men and women engaged in research should be as free as possible of the administrative duties involved in sponsored research. Therefore, the Bureau of Engineering Research (BER) was established and assigned the administrative responsibility for such research within the College.

The director of BER—himself a faculty member and researcher—maintains familiarity with the support requirements of both proposals and research in progress. He is aided by the College of Engineering Research Committee which is made up of faculty representatives from the academic departments of the College. This committee serves to inform BER of the needs and perspectives of the research program.

In addition to administrative support, BER is charged with providing certain technical assistance. Because it is not practical for each department to become self-sufficient in all phases of the supporting technology essential to present-day research, BER makes services available through support groups such as the machine shop, the electronics shop, and publication services.

FINAL REPORT

on

CONTRACT NO. NAS8-33430

DEVELOPMENT OF MATHEMATICAL MODELS FOR SOLID STATE SWITCHING DEVICES

Prepared for

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Marshall Space Flight Center, AL 35812

Prepared by

W. D. Raburn, Principal Investigator
Associate Professor of Electrical Engineering
P. O. Box 6169
The University of Alabama
University, Alabama 35486
and
J. C. Kim, Research Associate

Submitted by

The University of Alabama
P. O. Box 2846
University, Alabama 35486

January, 1980

BER REPORT NO. 249-25

DEVELOPMENT OF MATHEMATICAL MODELS FOR
SOLID STATE SWITCHING DEVICES

ABSTRACT

Models are developed for two types of remote power controllers (RPC). One type made by Westinghouse Electric Corporation (Westinghouse) and the other type made by Space Craft Incorporated (SCI). The models give the equations for the currents and voltages for all elements of passive loads as a function of time for both turn-on and turn-off. It is shown that the RPC can be considered as a combination of current and voltage sources. Equations are given for these sources which are essentially independent of the load being turned on and off. Experimental results are given for several types of loads and comparisons are made with the results obtained using the models.

INTRODUCTION

There has been considerable effort devoted to developing solid state switches for use in RPC. In particular Westinghouse and SCI have developed devices under contract with Marshall Space Flight Center intended to turn on and off a large range of passive loads. This report gives the results of the analyses of the RPC. It was at first assumed that the characteristics of some of the individual transistors used in RPC would have to be incorporated in the models. However after receiving the RPC from NASA and testing under several different loads it was determined that the passive components controlled the time response and the active components were only acting as current sources or saturated switches. Hence, the RPC would always act as a current source or a constant voltage source. One significant clue to this was that the time constants associated with turn-on and turn-off are always in the millisecond range. Switching times of transistors are usually less than a microsecond.

Basically the RPC consist of a power transistor for the output stage (there may be several transistors in parallel). The transistor is driven by a current source which is controlled by a R-C time constant circuit. If the output transistor is not saturated the load current will be proportional to the control current. If the output transistor is saturated the output voltage will be constant. The RPC usually also has protective circuits to prevent excessive power dissipation.

The models for the RPC were derived after experimentally observing the current and voltage wave forms for several different types of loads. It was found that the total load current was essentially independent of the type of load or value of the components during turn-on and before the output transistor saturates. Hence the RPC acts as a current source

during this time. Once the output transistor saturates the load voltage remains constant and the load current is determined by the components of the particular load. The same general analysis holds during turn-off.

Having decided on the form of the model for the RPC, a resistive load was selected and used to obtain the parameters for the sources used in the models. The voltage and current wave forms were photographed for turn-on and turn-off and the equations were fitted to the curves. A photograph showing such a curve is given in Figure 1. Then resistive-capacitive loads and resistive-inductive loads were used and experimental results were compared with results obtained using the models. The results were very good.

Models for both the Westinghouse and SCI RPC are presented. Results for loads using combinations of resistors, capacitors and inductors are given. Photographs showing experimental results are given and comparisons are made of results using the models and experimental results. First

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR

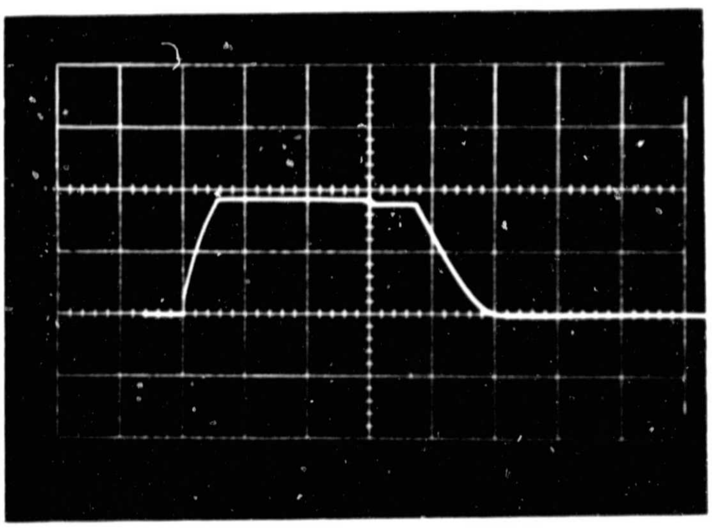


FIGURE 1. Load voltage waveform for resistive load of Westinghouse RPC.

the model for the Westinghouse RPC will be presented. Equations for the currents and voltages in each component will be given and then they will be compared with experimental results. Then the same procedure will be used to analyze the SCI device.

WESTINGHOUSE RPC

The Westinghouse RPC that were analyzed used essentially one design with several different current ratings. Two of the devices were used in this analysis, a five ampere device and a twenty ampere device. Both devices can be represented by the general model shown in Figure 2. The current source $i_a(t)$ is described according to Equation (1). However it can be very closely approximated by a simple exponential and the resulting equations are appreciably simplified. Hence the development is first made using Equation (1) and then the simplification is given.

When the RPC is turned on it will first act as a current source (position 1 of Figure 2) that supplies current to the passive load essentially independent of the elements of the load. This condition will hold until the switch saturates. The switch saturates when the load voltage reaches $V_L{}^1$ which is the applied voltage minus the saturated switch drop of about one half volt.

When the load voltage reaches $V_L{}^1$, S will switch to position 2 and the load voltage will remain at $V_L{}^1$. The load current will be determined by the load parameters and the conditions at switching. The steady state conditions will then be reached according to time constants determined by the load. However if the load current remains above a certain value (approximately 150 percent of rated current) for a period of time (approximately two seconds) a tripping mechanism will disconnect the load.

When the RPC is turned off by the controller the load voltage will

see an immediate small drop. This is represented in the model of Figure 2 as switching of S from position 2 to position 3. The load voltage will remain at a constant value V_L^2 for a time, t_s, which is a weak function of the value of load current at switching. After t_s the RPC will again act as a current source. This is represented in Figure 2 with S switching from position 3 to position 4.

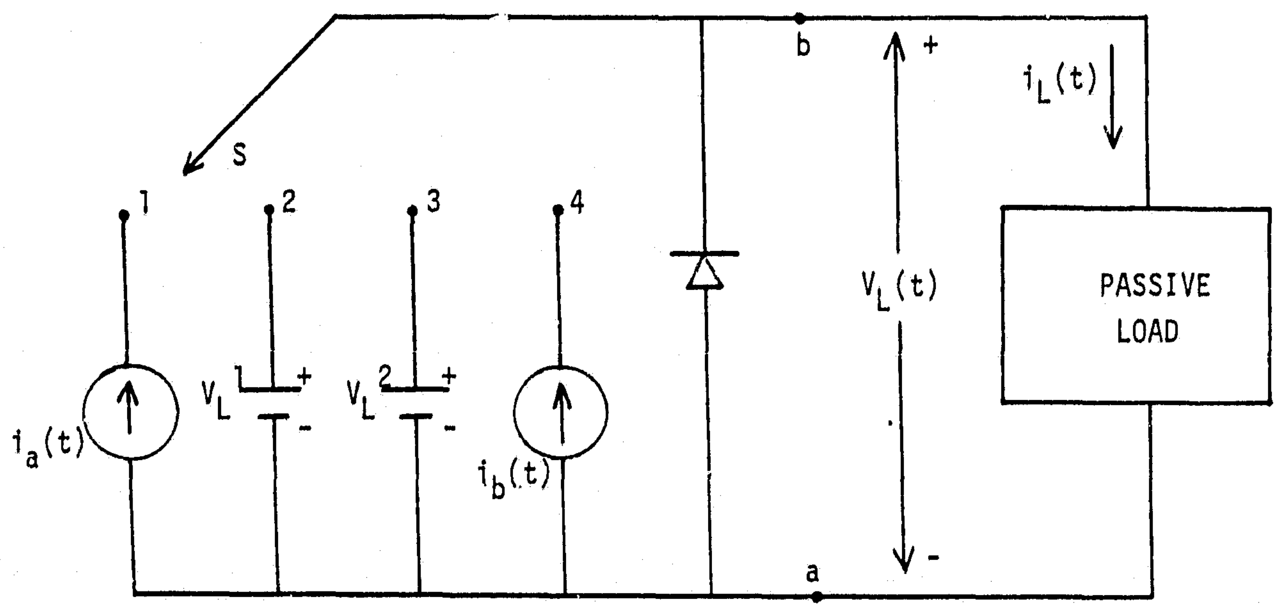


FIGURE 2. MODEL FOR WESTINGHOUSE RPC

The model of Figure 2 contains a diode across the load. Usually this diode is reverse biased and can be ignored. However for certain inductive loads it is possible for the load voltage to go negative. When the load voltage goes negative the diode starts to conduct and the load current is no longer equal i_b of Figure 2. This condition has been considered in analyzing turn-off of inductive loads. The model will now be used to analyze the turn-on and turn-off of certain loads. Experimental results will be given for comparison. However,

the equations of the current and voltages of Figure 1 will first be given. They are:

$$i_a(t) = \frac{I_1}{T_1} [t\ U(t) - (t - T_1)\ U(t - T_1)]$$

$$+ I_2 [1 - e^{-\alpha(t - T_1)}]\ U(t - T_1)\ , \qquad (1)$$

$$V_L^{\,1} = V_a - V_{ss}^{\,1} \approx V_a - 0.5 \text{ Volts}\ , \qquad (2)$$

$$V_L^{\,2} = V_a - V_{ss}^{\,2} \approx V_a - 1.5 \text{ Volts}\ , \qquad (3)$$

and

$$i_b(t) = I_3\ e^{-\frac{t - t_s}{\tau}}\ U(t - t_s)\ . \qquad (4)$$

R-C LOAD

The circuit of Figure 3 will be used to analyze a general R-C load. When the RPC is first turned on $i_L(t)$ will equal $i_a(t)$ of Equation 1. Of course $i_L = i_1 + i_2$. The load voltage will be given by,

$$V_L = \frac{I_1 R_2}{T_1} [\{t - R_2 C(1 - e^{-\beta t})\}\ U(t) - \{t - T_1 - R_2 C(1 - e^{-\beta(t - T_1)})\}\ U(t - T_1)]$$

$$+ R_2 I_2 [1 + \frac{1}{1 - \alpha/\beta} \{\alpha C R_2\ e^{-\beta(t - T_1)} - (1 - \alpha C R_1)\ e^{-\alpha(t - T_1)}\}]\ U(t - T_1)$$

(5)

where $\beta = \frac{1}{C(R_1 + R_2)}$.

At time, T_2, that S switches from position 1 to position 2 is found by the relation,

$$V_L(T_2) = V_L^{\,1}\ . \qquad (6)$$

After T_2 the voltage, V_L, across the load remains at V_L^1 and the current is given by

$$i_L(t) = \left[\frac{V_L^1}{R_2} + \left(\frac{V_L^1 - V_C(T_2)}{R_1}\right) e^{-\frac{(t - T_2)}{R_1 C}}\right] U(t - T_2) \qquad (7)$$

where $V_C(T_2)$ is the capacitor voltage at T_2 and is given by,

$$V_C(T_2) = (I_1 + I_2)\, R_2 + \frac{I_1 R_2 C(R_1 + R_2)}{T_1} \left(e^{-\beta T_2} - e^{-\beta(T_2 - T_1)}\right)$$

$$+ \frac{1}{1 - \alpha/\beta} \{\alpha C I_2 R_2 (R_1 + R_2)\, e^{-\beta(T_2 - T_1)} - e^{-\alpha(T_2 - T_1)}\}\ . \qquad (8)$$

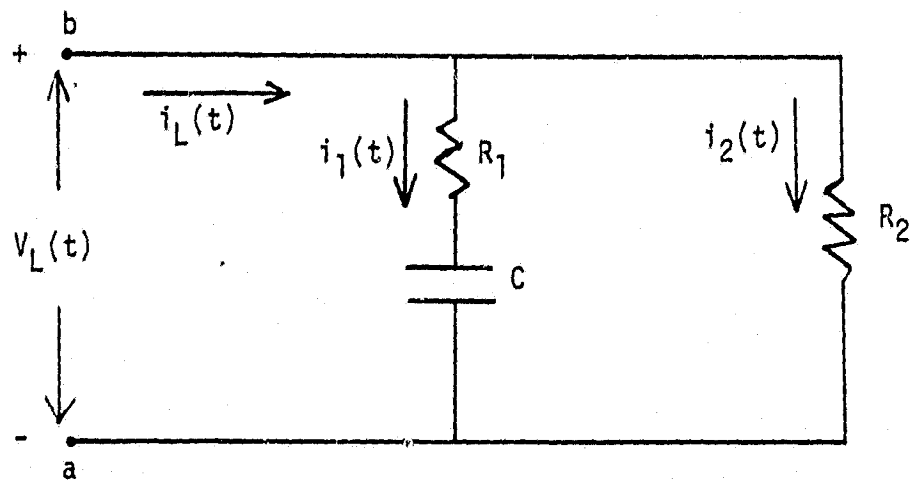


FIGURE 3. R-C LOAD USED IN ANALYSIS

For the turn-off assume that a steady state is reached and the RPC is turned off at t = 0. S is then moved from position 2 to position 3. At switching the voltage across the capacitor will be equal to $V_L^1 > V_L^2$ and hence, $i_1(t)$ will at first be negative and the total load current $i_L(t)$ will be less than V_L^2/R_2. The total load current for the range

$0 \leq t \leq t_s$ will be given by,

$$i_L(t) = \frac{V_L^{\ 2}}{R_2} - \left(\frac{V_L^{\ 1} - V_L^{\ 2}}{R_1}\right) e^{-\ t/R_1C} . \tag{9}$$

This equation holds until $t = t_s$. Then

$$i_L(t_s) = \frac{V_L^{\ 2}}{R_2} - \left(\frac{V_L^{\ 1} - V_L^{\ 2}}{R_1}\right) e^{-t_s/R_1C} \equiv I_3 . \tag{10}$$

At $t = t_s$ the S switches from position 3 to position 4, and

$$i_L(t) = I_3\ e^{-\ (t - t_s)/\tau}\ U(t - t_s) . \tag{11}$$

The load voltage is given by,

$$V_L(t) = \left[\frac{R_2 I_3}{C(R_1 + R_2) - \tau} \left\{(R_1C - \tau)\ e^{-\ (t - t_s)/\tau} + \frac{2R_2}{R_1 + R_2}\ e^{-\beta(t - t_s)}\right\}\right.$$
$$\left. + \frac{R_2 V_C(T_3)}{R_1 + R_2}\ e^{-\beta(t - t_s)}\right] U(t - t_s) . \tag{12}$$

If the current is approximated by a simple exponential during turn-on the analyses above are changed only for the currents and voltages during the rise time. These corresponding equations become,

$$i_a(t) = I_A(1 - e^{-\alpha t}) = i_L(t) , \tag{1-a}$$

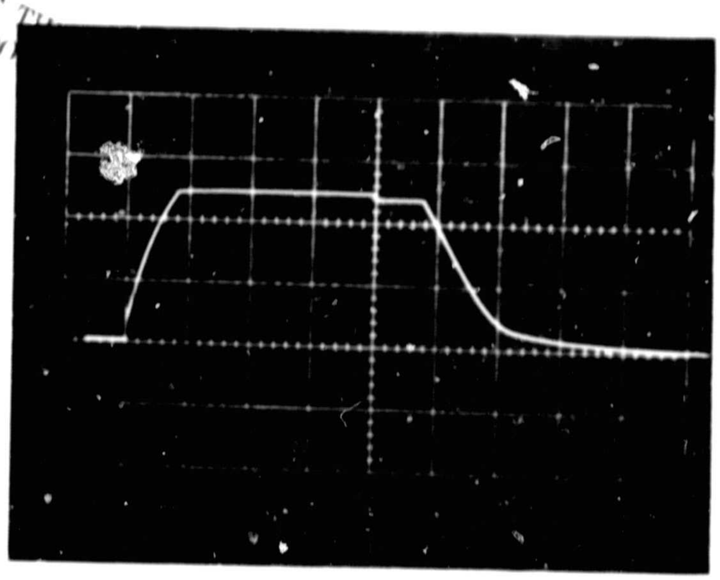
and

$$V_L(t) = I_A R_2\ [1 + \frac{1}{1 - \alpha/\beta} (\alpha R_2 C\ e^{-\beta t} - (1 - \alpha R_1 C)\ e^{-\alpha t})] . \tag{5-a}$$

The above set of equations can be used to calculate the currents and voltages for any Westinghouse RPC. Figures 4 and 5 show the experimental

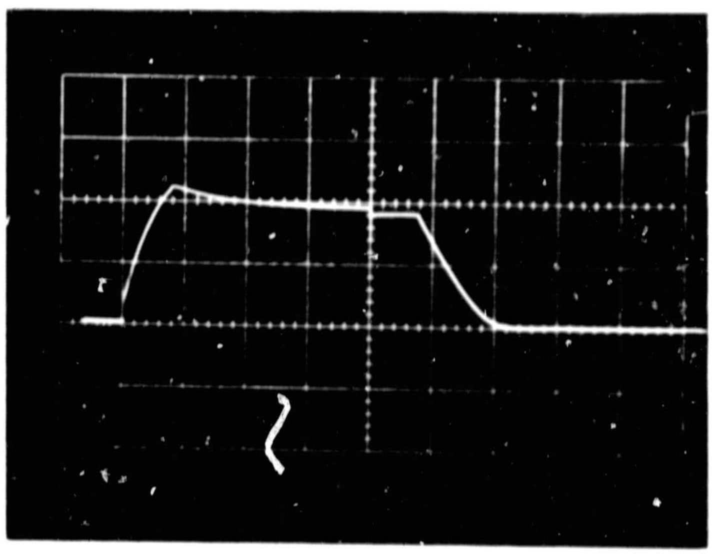
REPRODUCIBILITY OF TH
ORIGINAL PAGE IS PO

Potential - 1C ts/div.



(a)

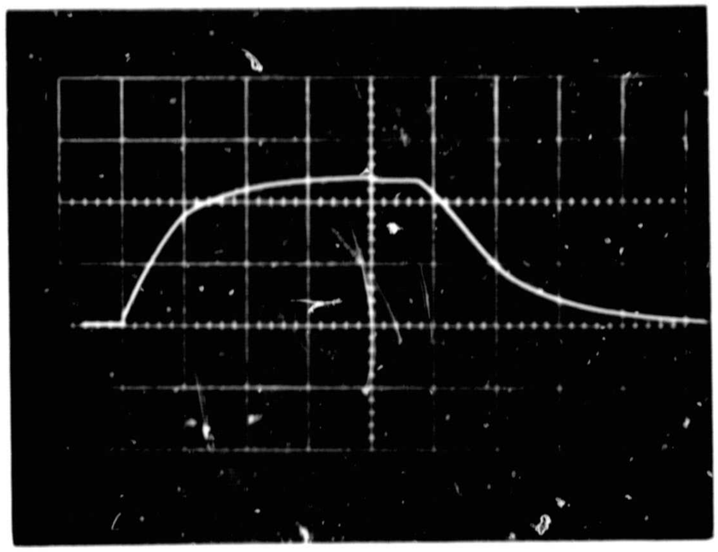
Current - 10 Amperes/div.



(b)

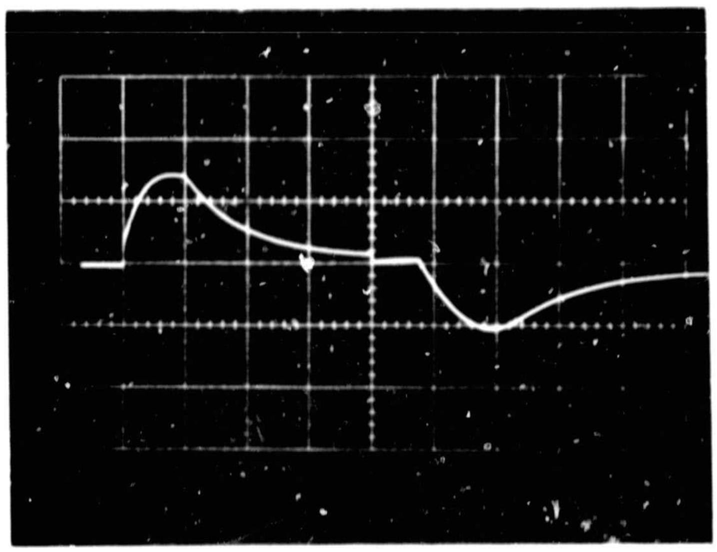
FIGURE 4. Total load voltage and current for R-C load of Figure 3. Horizontal time scale is 2 millisec./div.

Potential - 10 Volts/div.



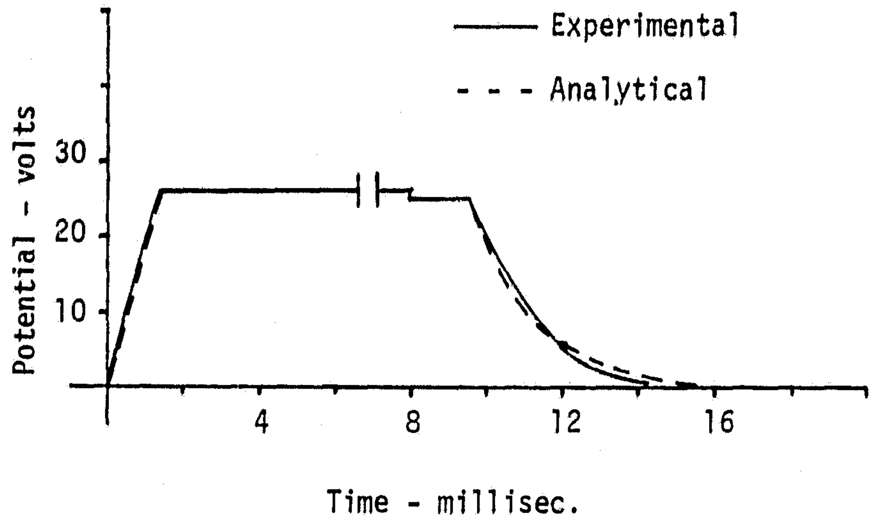
(a)

Potential - 5 Volts/div.



(b)

FIGURE 5. Capacitor voltage and R_1 voltage or capacitor current for load of Figure 3. Horizontal scale 2 millisec./div.



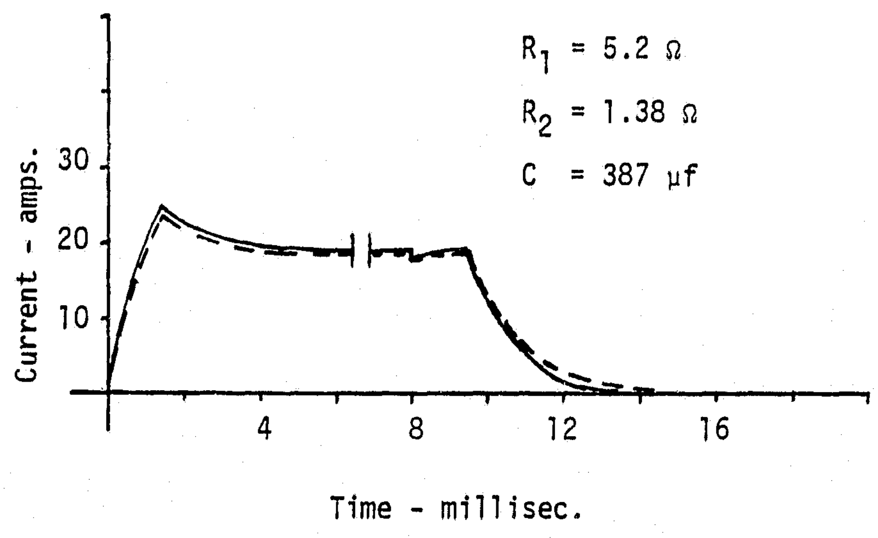


FIGURE 6. Comparison of experimental and analytical results for R-C load.

results for the twenty ampere device. The parameters used for these figures were $R_1 = 7.2\ \Omega$, $C = 387\ \mu f$, $R_2 = 1.38\ \Omega$. Figure 6 compares the total load current and voltage obtained experimentally with those calculated using the analytical model. Equation (1-a) was used for the current source during turn-on. The parameters used were $I_A = 31$ amperes, $\alpha = 0.9$/millisec., $t_s = t_o - K_1 I_3$ (where $t_o = 1.86$ millisec. and $K_1 = 0.027$ millisec./ampere) $\tau = \tau_o + K_2 I_3$ (where $\tau_o = 0.39$ millisec. and $K_2 = 0.076$ millisec./ampere). The values of the load parameters are given on Figure 6.

R-L LOAD

The R-L load is analyzed the same as the R-C load. For the notation refer to Figure 7. For turn-on, the load current is given by equation (1) for load voltages less than $V_L{}^1$. The corresponding load voltage is given by

$$
\begin{aligned}
V_L(t) = {} & \frac{I_1}{T_1} \Big[\Big\{ \frac{R_2{}^2}{\beta(R_1 + R_2)} (1 - e^{-\beta t}) + \frac{R_1 R_2}{R_1 + R_2} t \Big\} U(t) - \Big\{ \frac{R_2{}^2}{\beta(R_1 + R_2)} \\
& \times (1 - e^{-\beta(t - T_1)}) + \frac{R_1 R_2}{R_1 + R_2} (t - T_1) \Big\} U(t - T_1) \Big] \\
& + I_2 \Big[\frac{R_1 R_2}{R_1 + R_2} - \frac{\alpha R_2{}^2}{(R_1 + R_2)(\beta - \alpha)} e^{-\beta(t - T_1)} \\
& - \frac{R_2(R_1 - \alpha L)}{L(\beta - \alpha)} e^{-\alpha(t - T_1)} \Big] U(t - T_1)
\end{aligned}
\tag{13}
$$

where $\beta = \dfrac{R_1 + R_2}{L}$.

If Equation (1-a) instead of Equation (1) is used for the current

during the rise the load voltaoes reduces to

$$V_L(t) = I_A R_2 \left[\frac{R_1}{R_1 + R_2} - \frac{\alpha R_2}{(R_1 + R_2)(\beta - \alpha)} e^{-\beta t} - \frac{R_1 - \alpha L}{L(\beta - \alpha)} e^{-\alpha t}\right] \qquad (13\text{-a})$$

Equation (13) is monitored to determine the time T_2 that $V_L(t) = V_L^{\,1}$. The load voltage will remain at $V_L^{\,1}$ for $t > T_2$. To determine $i_L(t)$ for $t > T_2$ the value $I_o = i_1(T_2) = i_L(T_2) - V_L^{\,1}/R_2$ is first found. It is given by,

$$\begin{aligned} I_o = i_1(T_2) = \frac{I_1}{T_1} \Big[\Big\{&\frac{R_2}{R_1 + R_2} T_2 - \frac{R_2}{\beta(R_1 + R_2)} (1 - e^{-\beta T_2})\Big\} \\ &+ \Big\{\frac{R_2}{\beta(R_1 + R_2)} (1 - e^{-\beta(T_2 - T_1)}) - \frac{R_2}{R_1 + R_2} (T_2 - T_1)\Big\}\Big] \\ &+ I_2 \left[\frac{R_2}{R_1 + R_2} - \frac{L\beta - R_1}{L(\beta - \alpha)} e^{-\alpha(T_2 - T_1)} + \frac{\alpha R_2\, e^{-\beta(T_2 - T_1)}}{(R_1 + R_2)(\beta - \alpha)}\right. . \end{aligned} \qquad (14)$$

When using Equation (1-a)

$$I_o = I_A \left[1 - e^{-\alpha T_2}\right] - \frac{V_L^{\,1}}{R_2} \qquad (14\text{-a})$$

then the load current is,

$$i_L(t) = \left\{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_L^{\,1} + \left(I_o - \frac{V_L^{\,1}}{R_1}\right) e^{-\frac{R_1}{L}(t - T_2)}\right\} U(t - T_2) \quad . \qquad (15)$$

For the turn-off the load voltage will again be $V_L^{\,2}$ for $t < t_s$ and the load current for this time will be

$$i_L(t) = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_L^{\,2} + \left[\frac{V_L^{\,1} - V_L^{\,2}}{R_1}\right] e^{-\frac{R_1}{L} t} \quad . \qquad (16)$$

At $t = t_s$ the switch comes out of saturation and the load current is given by Equation (4). To find the value of $i_1(t_s)$, the value of current through the inductor at switching is monitored. It is given by,

$$I^o = i_1(t_s) = i_L(t_s) - \frac{V_L^{\,2}}{R_2} \quad . \tag{17}$$

The load voltage is then given by,

$$V_L(t) = \frac{I_3}{\tau(R_1 + R_2) - L}\{R_2(\tau R_1 - L)\, e^{-(t - t_s)/\tau} + \tau R_2^2\, e^{-\beta(t - t_s)}\}\, U(t - t_s)$$

$$- R_2 I_o^{\,o}(T_3)\, e^{-\beta(t - t_s)}\, U(t - t_s) \tag{18}$$

The above analysis is complete except for certain R-L loads that will drive the load voltage negative. If the load voltage goes more negative than the built-in voltage of the diode (few tenths of a volt) the diode will start to conduct and must be included in the analysis. For this analysis assume

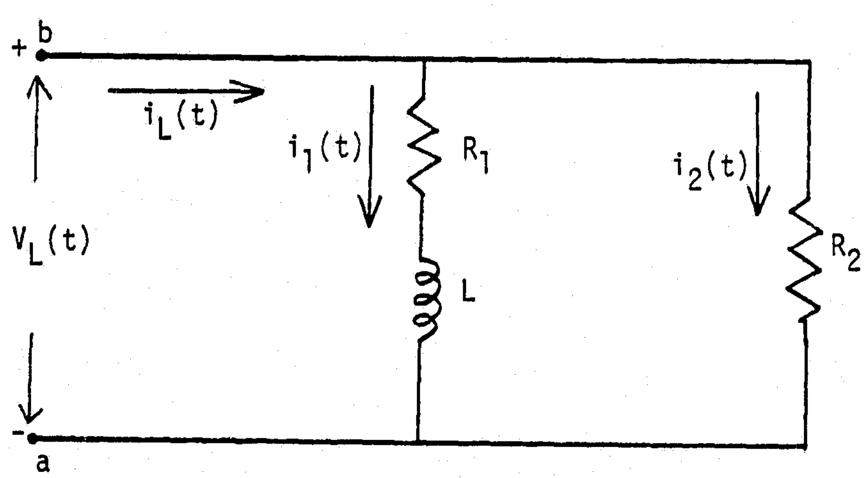


FIGURE 7. RESISTIVE-INDUCTIVE LOAD

$V_L(t) = - V_{bi}$ at $t = T_4$. The value of T_4 can be determined from equation (18). Again the value of current through the inductor at T_4 must be determined. This value is given by,

$$I_o^{o} = i_1(T_4) = i_L(T_4) + \frac{V_{bi}}{R_2} \quad . \tag{19}$$

Assuming that the diode can be modeled with a resistance r_d the load voltage and current for $t > T_4$ are given by,

$$V_L(t) = \frac{I_3R_2}{\tau(R_2 + R_1) - L} \{(\tau R_1 - L)\, e^{-(t - t_s)/\tau} + \tau R_2\, e^{-\beta(t - t_s)}\}\, U(t - t_s)$$

$$- R_2 I_o^{o}(T_4)\, e^{-\beta(t - T_4)}\, U(t - T_4) \tag{20}$$

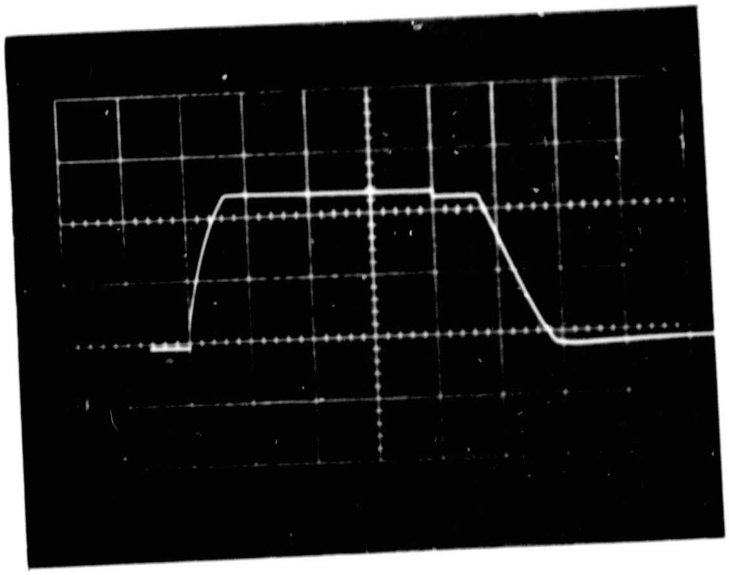
and

$$i_L(t) = \frac{R_2 I_o^{o}(T_4)}{r_d} e^{-\beta(t - T_4)}\, U(t - T_4) + I_3 \left[1 - \frac{R_2(\tau R_1 - L)}{r_d \{\tau(R + R_1) - L\}}\right]$$

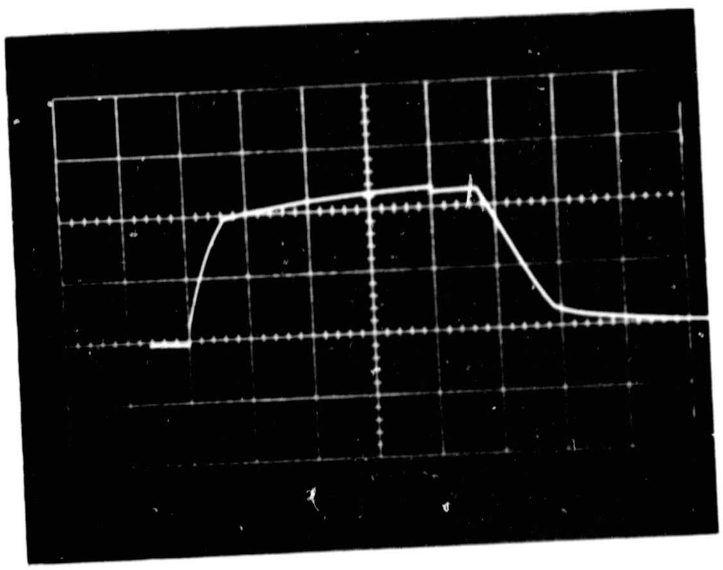
$$e^{-(t - t_s)/\tau}\, U(t - T_3) - \frac{I_3 \tau R_2^{2}}{r_d \{\tau(R + R_1) - L\}} e^{-\beta(t - t_s)}\, U(t - T_s) \quad . \tag{21}$$

Typical experimental results of the Westinghouse twenty ampere device are shown in Figures 8 and 9. The values of the load parameters used in these figures are $R_1 = 5.75\ \Omega$, $R_2 = 1.38\ \Omega$, $L = 20$ mh. Figure 10 is a comparison of the experimental results and the analytical results obtained using the above equations with Equation (1-a) being used for the turn-on current.

REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR



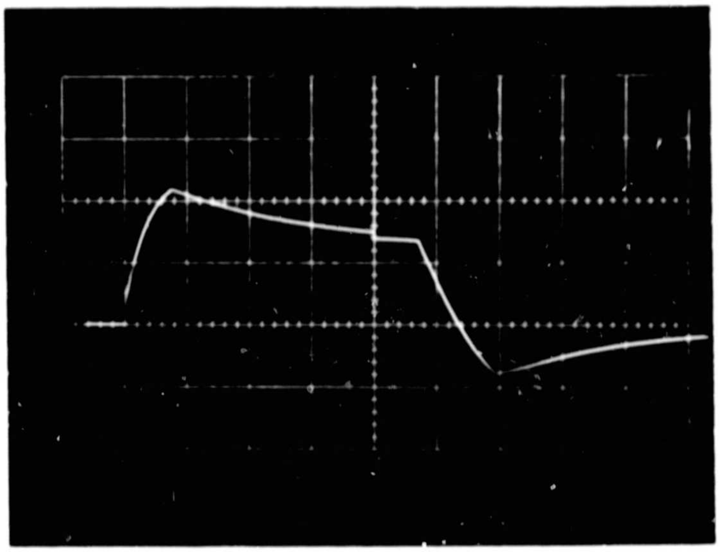
(a)



(b)

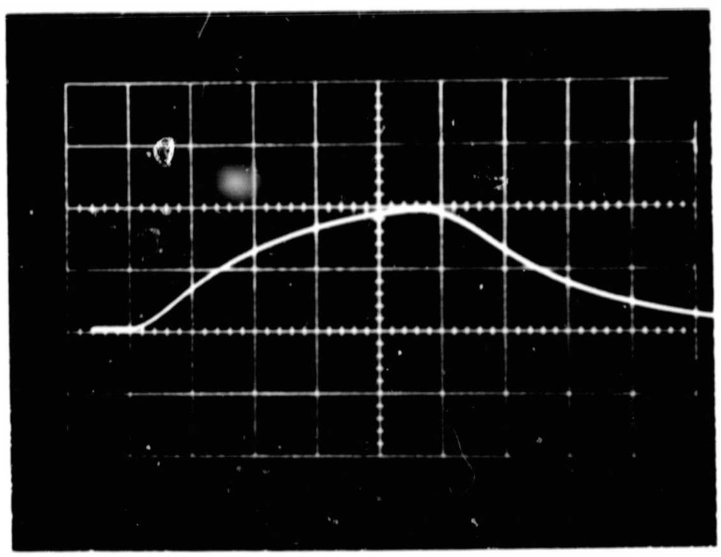
FIGURE 8. Total voltage and current waveform for the R-L load of Figure 7. Horizontal time scale is 2 millisec./div.

Potential - 10 volts/div.



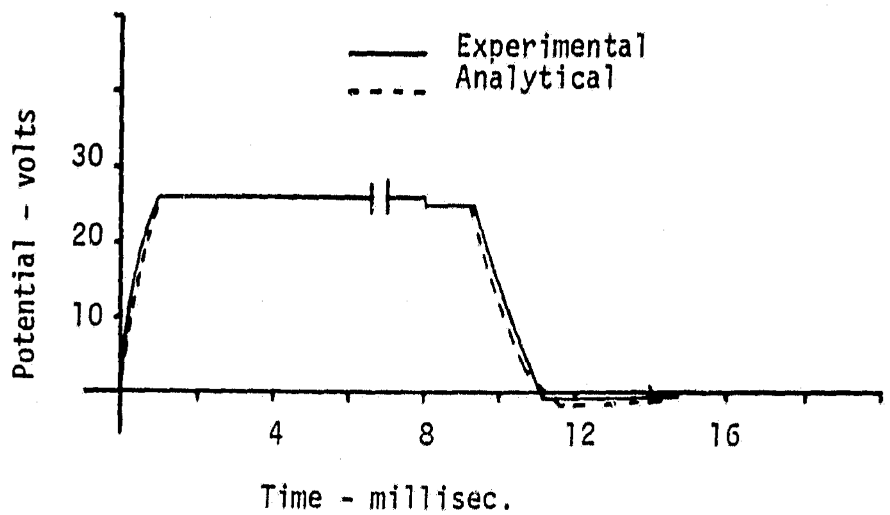
(a)

Current - 2.5 amperes/div.



(b)

FIGURE 9. Voltage and current waveform for inductor of Figure 7. Horizontal time scale is 2 millisec./div.



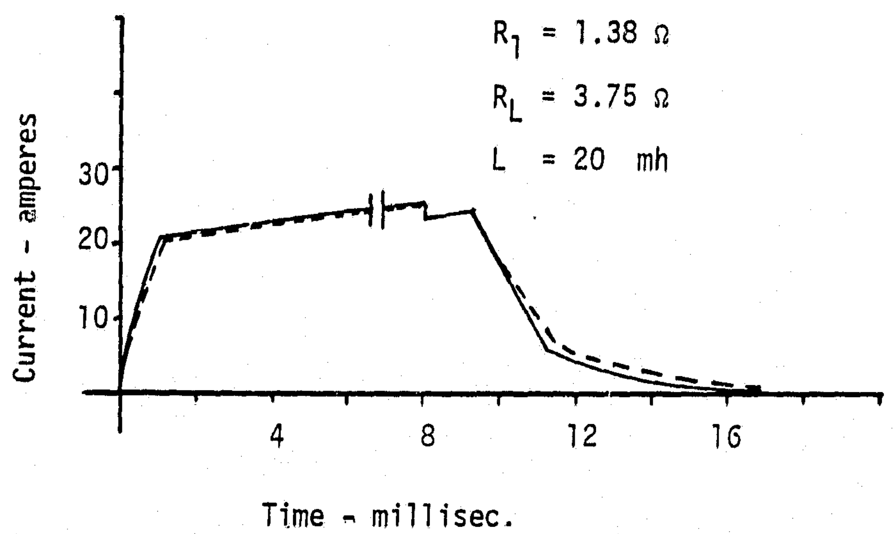


FIGURE 10. A comparison of experimental results and analytical results for total load voltage and current for a R-L load.

SCI MODEL

The analysis of the SCI RPC is similar to that of the Westinghouse device. However the behavior is quite different. The SCI device tends to act as a current source with a constant slope both during the turn-on and turn-off. There is a tendency to want to oscillate between the current mode and a current protecting mode for certain capacitive loads. However the oscillations are about the value predicted by the current mode and usually will not be detrimental to the load. The linear model for this case is represented by Figure 11. There is no diode across the output in the SCI device. Hence, there will usually be a large negative voltage spike across the output during turn-off.

The equations for the sources of Figure 11 are given by:

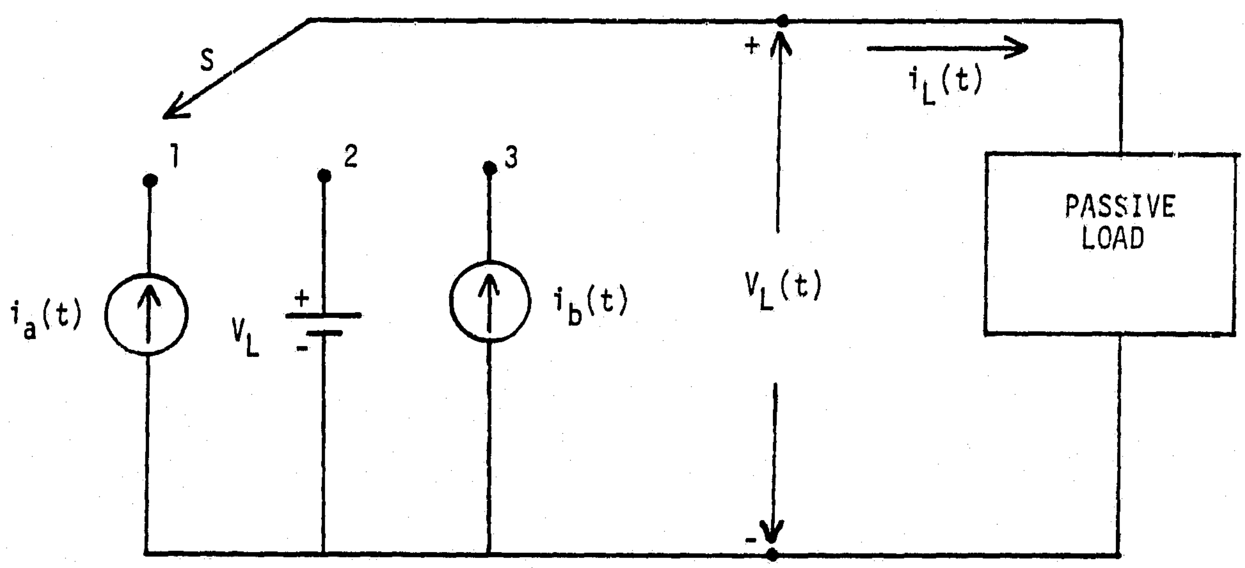


FIGURE 11. MODEL FOR SCI RPC

$$i_a(t) = \frac{I_1}{T_1} t\, U(t) - \frac{I_1}{T_1} (t - T_1)\, U(t - T_1) \,, \tag{22}$$

$$V_L = V_a - V_{ss} \quad , \tag{23}$$

$$i_b(t) = I_2 \left[1 - \frac{t}{T_2}\right] U(t) + I_2 \left(\frac{t - T_2}{T_2}\right) U(t - T_2) \ . \tag{24}$$

During turn on the load current $i_L(t)$ will equal $i_a(t)$ and $V_L(t)$ will be determined by the load. This will hold until $V_L(t)$ reaches V_L. $V_L(t)$ will then remain constant at V_L and the current $i_L(t)$ will be determined by the load. During turn-off the current $i_L(t) = i_b(t)$ and the voltage $V_L(t)$ is determined by the load and the initial conditions. Analysis will now be given for a R-C load and a R-L load.

R-C LOAD

The R-C load of Figure 3 is again used for this analysis. During turn-on the load current will initially be given by,

$$i_L(t) = i_a(t) = \frac{I_1}{T_1} \left[t \; U(t) - (t - T_1) \; U(t - T_1)\right] \quad . \tag{25}$$

As can be seen from Equation (25) the load current would be limited to I_1 at $t \geq T_1$. However, for most loads the output stage will saturate before $i_L(t) = I_1$. Since Equation (25) is only satisfied before saturation it can usually be described with only the first term, i.e.,

$$i_L(t) = \frac{I_1}{T_1} t \; U(t) \ . \tag{25-a}$$

For this case the load voltage is given by

$$V_L(t) = \frac{I_1}{T_1} \; R_2 \left[t - R_2 C(1 - e^{-\beta t})\right] \ . \tag{26}$$

Where

$$\beta = \frac{1}{(R_1 + R_2)C} \quad .$$

Also,

$$i_2(t) = \frac{V_L(t)}{R_2} = \frac{I_1}{T_1}\,[t - R_2C(1 - e^{-\beta t})] \ , \tag{27}$$

and

$$i_1(t) = i_L(t) - i_2(t) = \frac{I_1}{T_1}\,R_2C(1 - e^{-\beta t}) \ . \tag{28}$$

These relations will hold until $V_L(t) = V_L$. The time T_2 that this occurs can be determined from Equation (26). For $t > T_2$

$$V_L(t) = V_L \quad , \tag{29}$$

$$i_2(t) = V_L(t)/R_2 = V_L/R_2 \quad , \tag{30}$$

$$i_1(t) = \left(\frac{V_L - V_o}{R_1}\right) e^{-\frac{(t - T_2)}{R_1C}}\ U(t - T_2) \ . \tag{31}$$

Where,

$$V_o = V_C(T_2) = \frac{I_1}{T_1}\,R_2\,[T_2 - \frac{1 - e^{-\beta T_2}}{\beta}] \quad .$$

The total load current is

$$i_L(t) = i_1(t) + i_2(t) = \frac{V_L}{R_2} + \frac{V_L - V_o}{R_1}\, e^{-\frac{(t - T_2)}{R_1C}}\ U(t - T_2) \ . \tag{32}$$

For the turn-off it will be assumed that the RPC has been on for a long time and that the capacitor voltage has been charged to V_L. The total load current is then

$$i_L(t) = I_2 [1 - \frac{t}{T_3}] U(t) + I_2 [\frac{t - T_3}{T_3}] U(t - T_3) . \quad (33)$$

Where,

$$I_2 = V_L/R_2 .$$

The load voltage is

$$V_L(t) = V_L - \frac{V_L}{T_3} [t - R_2C(1 - e^{-\beta t})] U(t)$$
$$- \frac{V_L}{T_3} [(t - T_3) - R_2C(1 - e^{-\beta(t - T_3)}] U(t - T_3) , \quad (34)$$

and, of course, $i_2(t) = V_L(t)$ and $i_1(t) = i_L(t) - i_2(t)$.

Figure 12 gives the total load voltage and current for the five ampere SCI device and Figure 13 gives the capacitor voltage for the same conditions. The values of the load parameters used were $R_1 = 5.2\ \Omega$, $R_2 = 5.9\ \Omega$, $C = 554\ \mu f$.

R-L LOAD

Again for this analysis it is assumed that the output transistor saturates before the current is limited. Hence the load current is,

$$i_L(t) = \frac{I_1}{T_1}(t)\ U(t) . \quad (35)$$

The load voltage is,

$$V_L(t) = (\frac{I_1}{T_1})(\frac{R_2}{R_1 + R_2}) [R_1 t + \frac{R_2}{\beta}(1 - e^{-\beta t})] , \tag{36}$$

where the load of Figure 7 is assumed and,

$$\beta = \frac{R_1 + R_2}{L} .$$

This will hold to $t = T_2$ such that $V_L(T_2) = V_L$. For $t > T_2$ $V_L(t) = V_L$ and the load current is,

$$i_L(t) = V_L (\frac{1}{R_1} + \frac{1}{R_2}) + (I_o - \frac{V_L}{R_1}) e^{-\frac{R_1}{L} t} , \tag{37}$$

where $I_o = i_2(T_2)$, and is given by

$$I_o = I_1 (\frac{T_2}{T_1}) - \frac{V_L}{R_2} . \tag{38}$$

For the turn-off the total load current is given by equation (33). The load voltage is given by,

$$V_L(t) = V_L - V_L [(\frac{R_2}{R_1})(\frac{1}{\beta T_3})(1 - e^{-\beta t}) + \frac{t}{T_3}] U(t)$$

$$+ V_a [(\frac{R_2}{R_1})(\frac{1}{\beta T_3})(1 - e^{-\beta(t - T_3)}) + (\frac{t - T_3}{T_3})] U(t - T_3) . \tag{39}$$

The peak negative voltage will usually occur at approximately $t = T_3$. From Equation (39) this is given by

$$V_L(Neg) = V_L(T_3) = - (\frac{V_L}{\beta T_3})(\frac{R_2}{R_1}) . \tag{40}$$

If the branch currents are desired they are obtained by the simple relations $i_2(t) = V_L(t)/R_2$ and $i_1(t) = i_L(t) - i_2(t)$.

Figure 14 shows the total load voltage and current for this case. Figure 15 shows the inductor voltage and current. The values of the load parameters used were $R_1 = 9.15\ \Omega$, $R_2 = 5.2\ \Omega$, $L = 100$ mh. All of the curves were taken using the SCI five ampere device. The values of the parameters used in the model were:

$$\left(\frac{I_1}{T_1}\right) = 5.86\ \frac{\text{amp}}{\text{m sec.}}$$

$$T_3 = T_o + K\ I_2\quad,$$

where

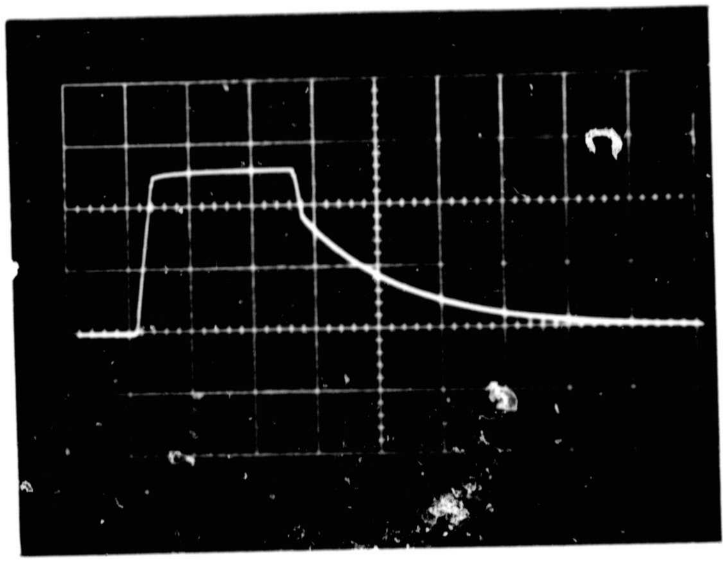
$T_o = 0.693$ m sec.,

$K = 0.067$ m sec/amp and I_2 is the value of $i_L(t)$ at $t = 0$ or the steady state load current.

Throughout the experimental work two fully charged Sears Die Hard auto batteries were connected in series and used as the supply voltage. A Hewlett-Packard 214A pulser was used as the control signal.

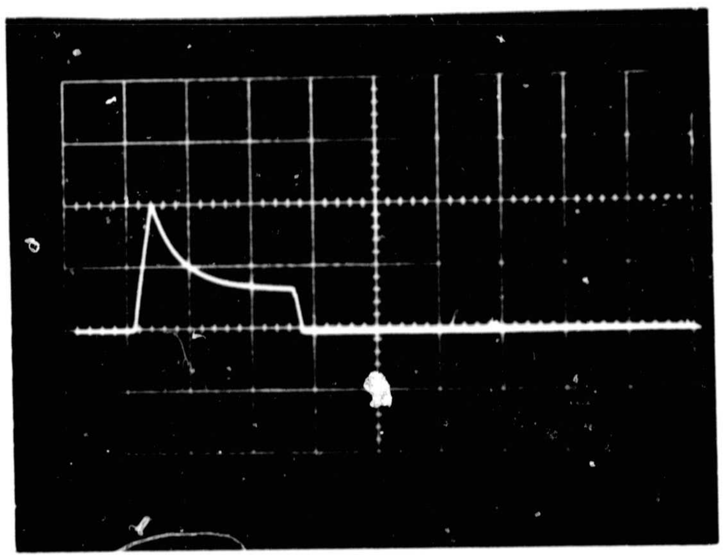
REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

Potential - 10 volts/div.



(a)

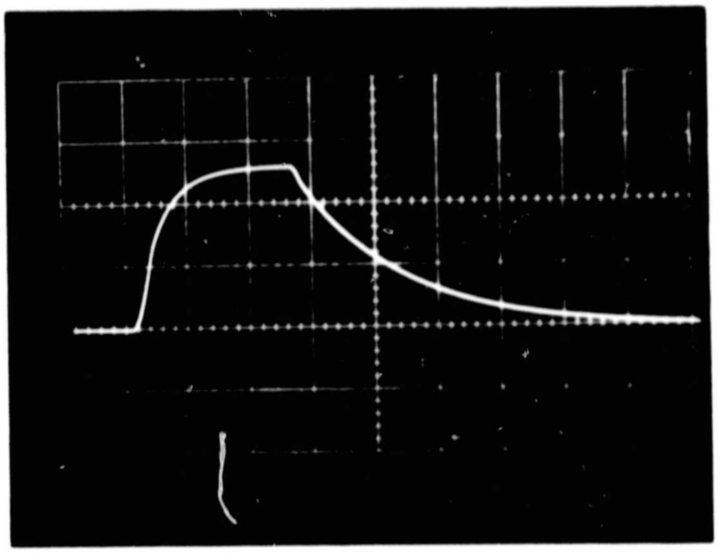
Current - 5 ampere/div.



(b)

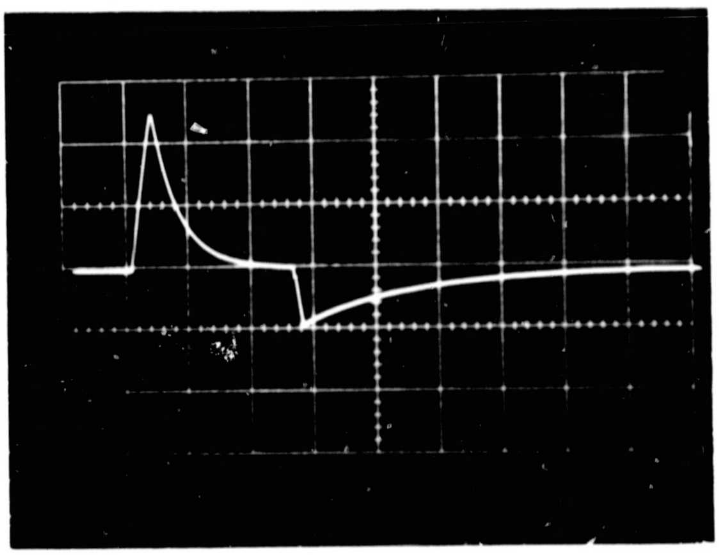
FIGURE 12. Total load voltage and current waveform for SCI RPC with R-C load. Horizontal time scale is 5 millisec./div.

Potential - 10 volts/div.



(a)

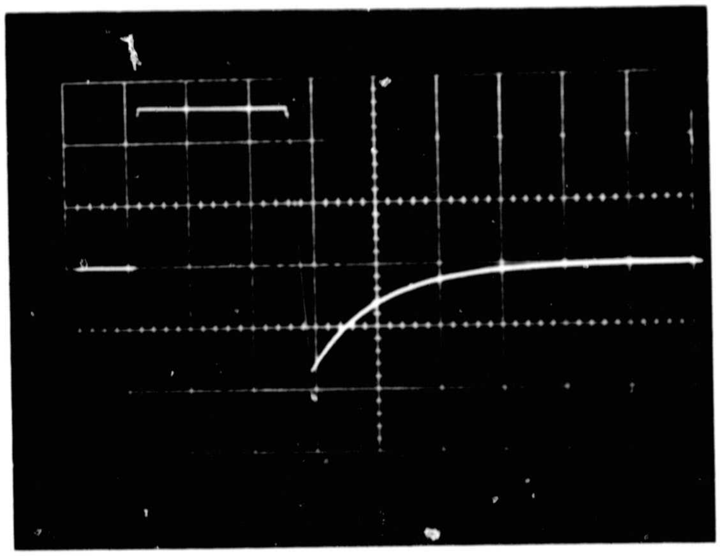
Current - 2.5 amperes/div.



(b)

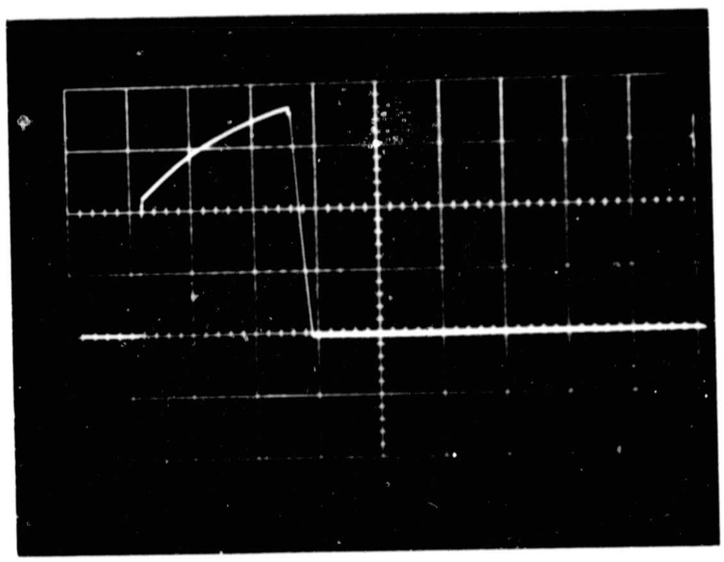
FIGURE 13. Capacitor voltage and current waveforms for SCI RPC with R-C load. Horizontal time scale is 5 millisec./div.

Potential - 10 volts/div.



(a)

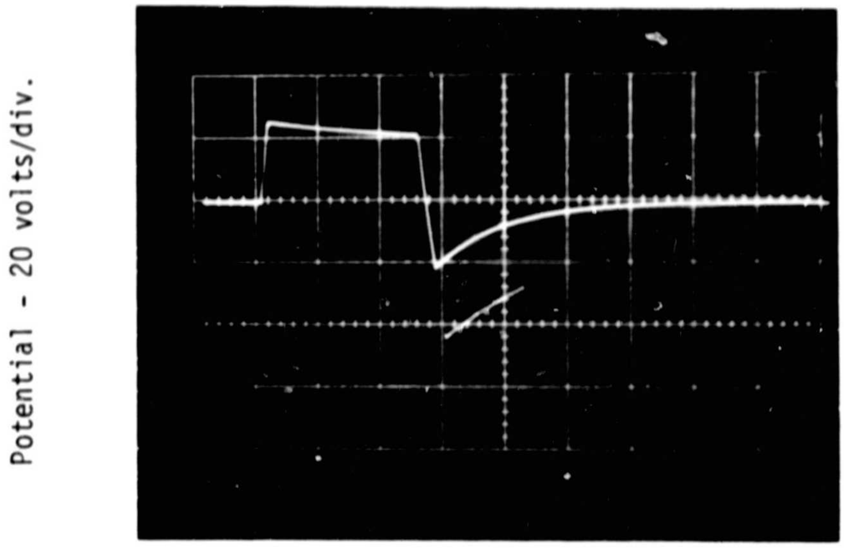
Current - 2 amperes/div.



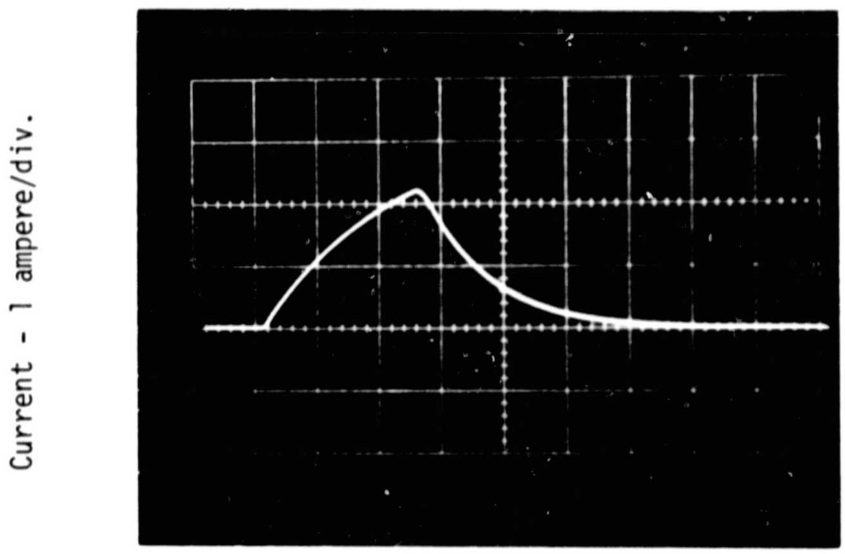
(b)

FIGURE 14. Total load voltage and current waveforms for SCI RPC with R-L load. Horizontal time scale is 5 millisec./div.

REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR



(a)



(b)

FIGURE 15. Inductor voltage and current waveforms for SCI RPC with R-L load. Horizontal time scale is 5 millisec./div.

THE UNIVERSITY OF ALABAMA
COLLEGE OF ENGINEERING

The College of Engineering at The University of Alabama has an undergraduate enrollment of more than 1,800 students and a graduate enrollment exceeding 100. There are approximately 100 faculty members, a significant number of whom conduct research in addition to teaching.

Research is an integral part of the educational program, and research interests of the faculty parallel academic specialities. A wide variety of projects are included in the overall research effort of the college, and these projects form a solid base for the graduate program which offers twelve different master's and five different doctor of philosophy degrees.

Other organizations on the University campus that contribute to particular research needs of the College of Engineering are the Charles L. Seebeck Computer Center, Geological Survey of Alabama, Marine Environmental Sciences Consortium, Mineral Resources Institute—State Mine Experiment Station, Mineral Resources Research Institute, Natural Resources Center, School of Mines and Energy Development, Tuscaloosa Metallurgy Research Center of the U. S. Bureau of Mines, and the Research Grants Committee.

This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of teaching, research, and public service.